When a System Breaks: A Queuing Theory Model for the Number of Intensive Care Beds Needed During the COVID-19 Pandemic

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Abstract

Due to some misconception over the number of intensive care beds that might be needed to care for COVID-19 patients we applied an exponential growth model to evaluate the preparedness of a typical urban intensive care unit to cope with possible demand for beds resulting from the COVID-19 pandemic. The model's predictions are broadly supported by data from Italy and suggest that Australian hospitals do not currently have the capacity to accommodate possible demand and, as a result, the future mortality rate may be much higher than expected.

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The Coronavirus 2019 (COVID-19) pandemic is currently pushing health systems to, and possibly beyond, their limits [1]. In Italy, the exponential rise in cases has required an exponential rise in health resource utilisation and in particular, an exponential demand for ICU beds [2]. In Lombardy, a relatively wealthy and well-resourced region of Italy, local authorities proposed building a 400-bed ICU field hospital in the Milan fairgrounds in a desperate attempt to meet this demand. The plan was rejected as it was impossible to staff and equip in time [3].

How many ICU beds will the pandemic require? To answer this question we propose a simple model based on an average ICU length of stay of 10 days and we then compare this model against recent real-world data from Lombardy [4]. It is important to note that the model itself is not important but reflects a realistic scenario and is demonstrated to be supported by international data.

Model

To begin, let's imagine we are in a tertiary hospital in an Australian capital city where there is a steady state of 20 new positive COVID-19 cases each and every day. Of these 20 new cases, 3 will need admission to a low acuity hospital ward and 1 will need ICU admission each day [5].

How many ICU beds do we require?

From queuing theory, Little's law describes the relationship between the number of patients in a system (L) and the average arrival rate (λ) and length of time (W) the patient spends in that system [6].

$$L = \lambda . W$$

$$L = 1\, bed/day \times 10\, days = 10\, beds$$

Now, instead of a steady state, imagine the number of total positive cases in the community increases by 20% every day (23% currently in Australia [7]). On the day you have 100 total positive cases, you will have approximately 120 the next. Those 20 new positive cases will require 1 new ICU admission and a 10 bed ICU to service that rate of admissions.

That implies that the number of ICU beds needed is approximately 10% the Total Positive Cases or 50% of the number of new positive cases. Australia has around 2,200 ICU beds, which implies if public health measures fail to curb the rate of growth, Australia's ICU capacity will be exceeded at around 22,000 COVID-19 cases sometime around the 5th April, 2020. Other sources [8] have suggested that Australia could cope with up to 44,580 COVID-19 cases, but even if this is true it only grants a 3 day extension to the 8th April, 2020. The practical impact on ICU capacity of this scenario is made clear in Figure 1.

Figure 1. Growth in ICU bed demand by time and proportion of positive cases requiring intensive care
[insert figure here]

Figure 1 illustrates that under this scenario a single hospital requires 31 ICU beds on day 7, and almost 200 on day 14. As a sensitivity analysis around the assumed proportion of new positive cases requiring an ICU bed we also considered half our assumed rate of new patients requiring an ICU bed (2.5%) but even in this more conservative assumption 16 beds are required on day 7 and 60 on day 14. If a more pessimistic proportion was to occur (10% of new cases requiring an ICU bed) the green line in Figure 1 indicates that any current hospital would be overwhelmed within several days. Further, we noted above that our assumption of a 20% daily increase in total positive cases is slightly conservative relative to the current rate in Australia of 23%. If we consider this graph as representing not a single hospital but all of Australia, as of 25th March, we are at Day 14.

Model Validation

This scenario is based on a model of constant exponential growth in disease cases. To evaluate how realistic this assumption is we evaluated exponential and linear growth models against a recent dataset from Lombardy [9], and compared the predicted to actual number of ICU patients in Figure 2.

Figure 2. ICU admission rate per 100,000 population in Lombardy initially increases exponentially followed by a steep linear increase

[insert figure here]

Based on piecewise regression models (Figure 2), it can be seen, for days 1-14 an exponential model correctly predicts the expected rise in the number of ICU patients (R^2=0.96), but from day 15 onwards, whilst ICU admissions continue to rise steeply, the number of patients increases instead in a linear fashion (R^2=0.99) with no evidence of exponential increase (p=0.5).

What happened at Day 15? To answer this question we then looked at the hospitalised patient ICU admission versus mortality rate in Figure 3 in which the blue line represents number of deaths ÷ number of hospital admissions and the red line represents number of ICU admissions ÷ number of hospital admissions.

Figure 3. ICU admission and death rates among hospitalized patients

[insert figure here]

In Figure 3 we found the mortality rate among hospitalized averaged 8.8% from Day 1 to Day 14 and was essentially steady (p=0.9), but from Day 15, the mortality rate dramatically rises (p<0.001) with an average mortality of 22.7% from day 15 onwards.

The authors' conjecture is that initially the 8.8% mortality is predominately from COVID-19 patients in ICU but from around Day 15 onwards, the increased demand for ICU beds outstrips the capacity of the system to supply them, and patients perish not from COVID-19 *per se* but from lack of access to an ICU bed. This is also illustrated in Figure 3 where the ICU admission rate falls as demand increases, with a corresponding increase in the mortality rate.

Finally, the capacity required to prevent the excess deaths after day 14 can be seen in Figure 4 which compares the observed number of deaths among hospitalized patients (green line), the number expected under the assumption of a constant rate (number of hospital admissions x 0.088, blue line) and under a piecewise regression with change in slope after Day 14 (red line). All three are consistent until day 14 but the observed and regression-predicted lines diverge from the constant rate line from Day 15 onwards.

Figure 4. Number of deaths observed and expected under a) a constant mortality rate and b) a model of change in rate after day 14

[insert figure here]

These data imply that the eventual mortality rate of COVID-19 may be much higher than currently estimated because once the system reaches breaking point and there are insufficient ICU Beds, mortality rises dramatically.

Conclusion

While the specific form of the proposed model can be debated, it does appear to represent a realistic clinical scenario, is consistent with international data and suggests the conclusion that the impending demand for ICU beds could overwhelm capacity in even the largest Australian hospitals in the near future. Australia must immediately take all available measures to rapidly decrease the rate of new cases and radically increase the number of ICU beds otherwise we may face the same fate as Italy, or worse.

References

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9) Data accessed from: https://www.kaggle.com/sudalairajkumar/covid19-in-italy#covid19_italy_region.csv

Figure 1

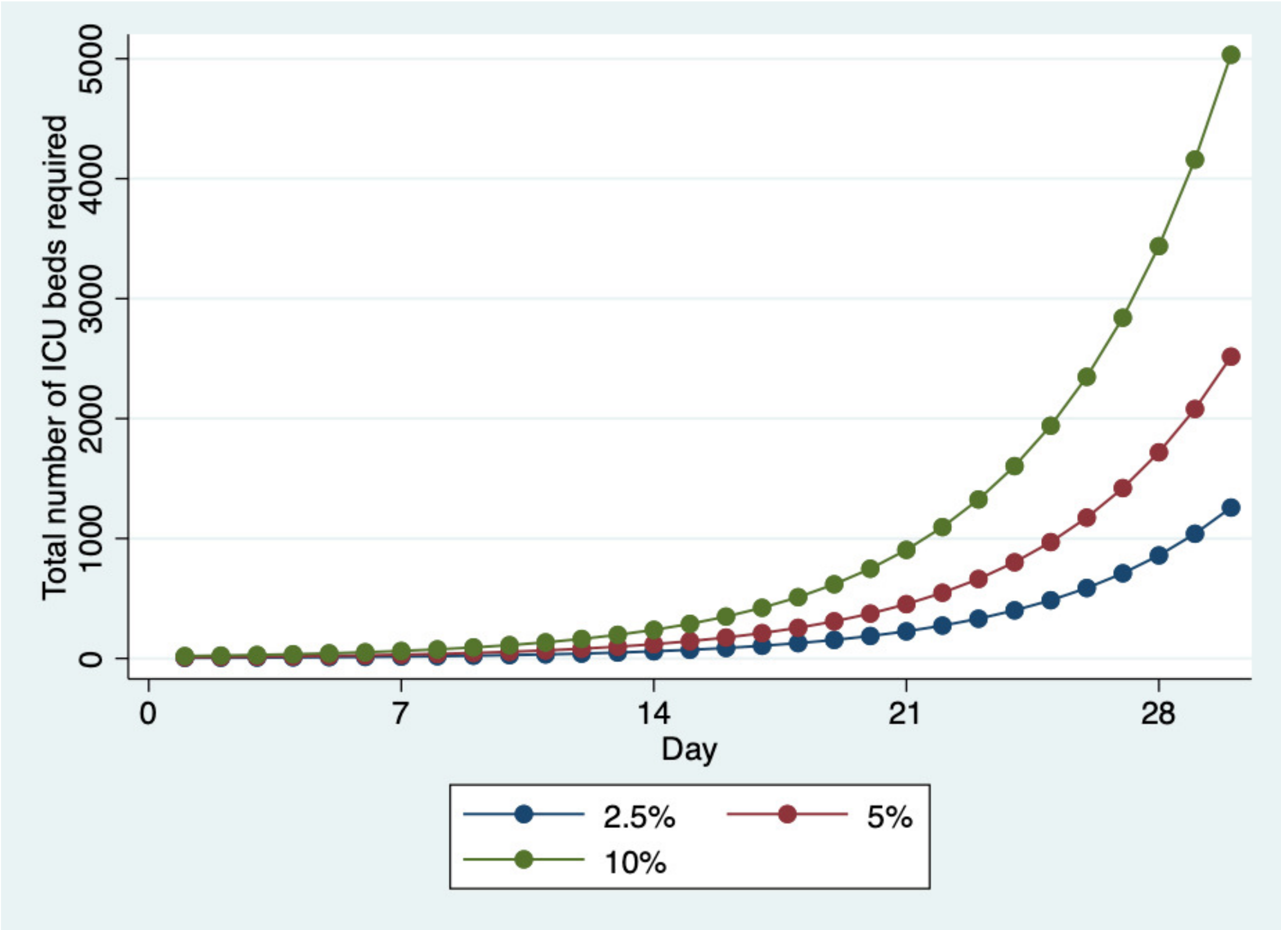

Total number of ICU beds required
0
1000
2000
3000
4000
5000
0
7
14
21
28
Day
2.5%
5%
10%

Figure 2

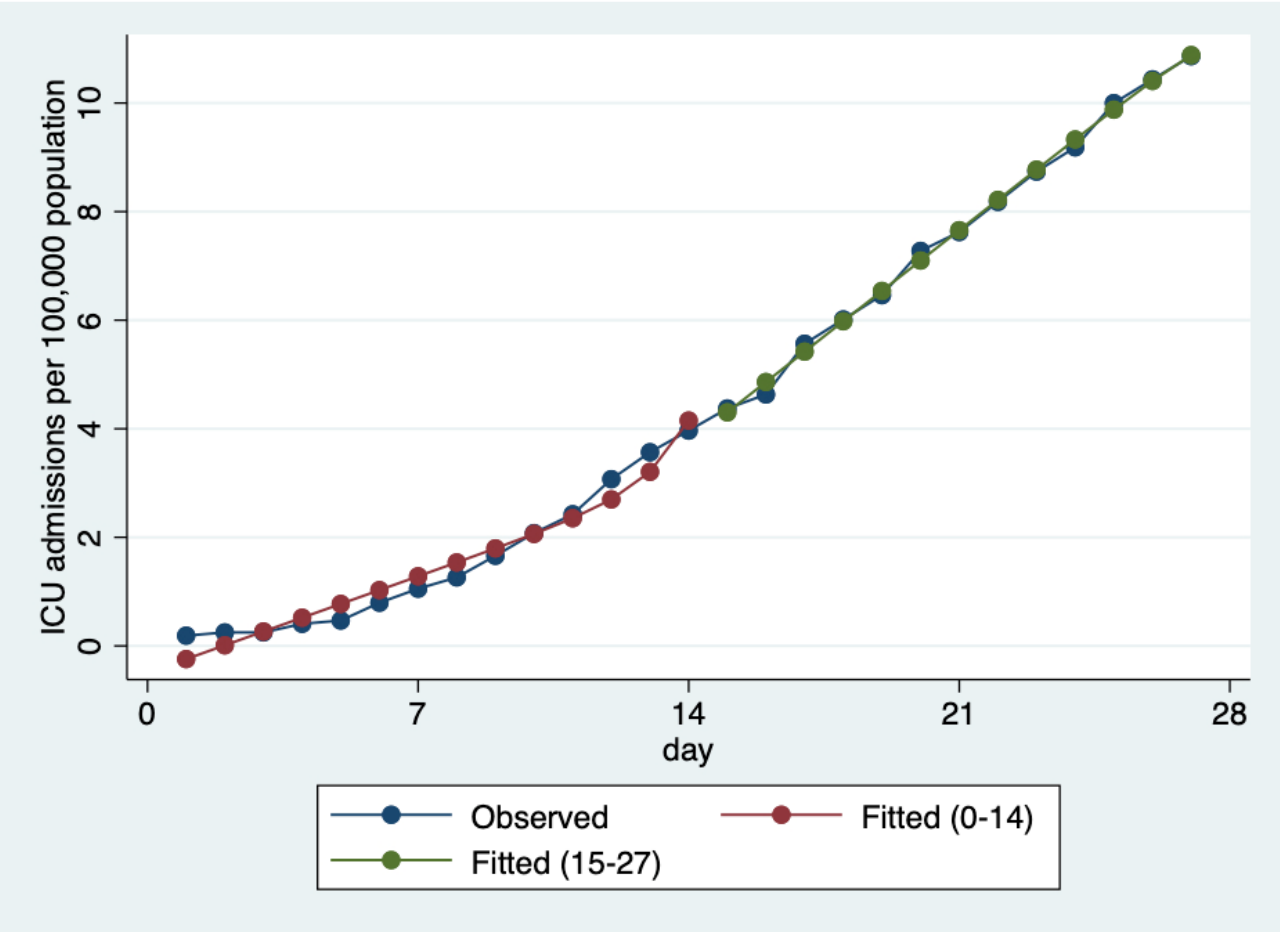

ICU admissions per 100,000 population
0
2
4
6
8
10
0
7
14
21
28
day
Observed
Fitted (0-14)
Fitted (15-27)

Figure 3

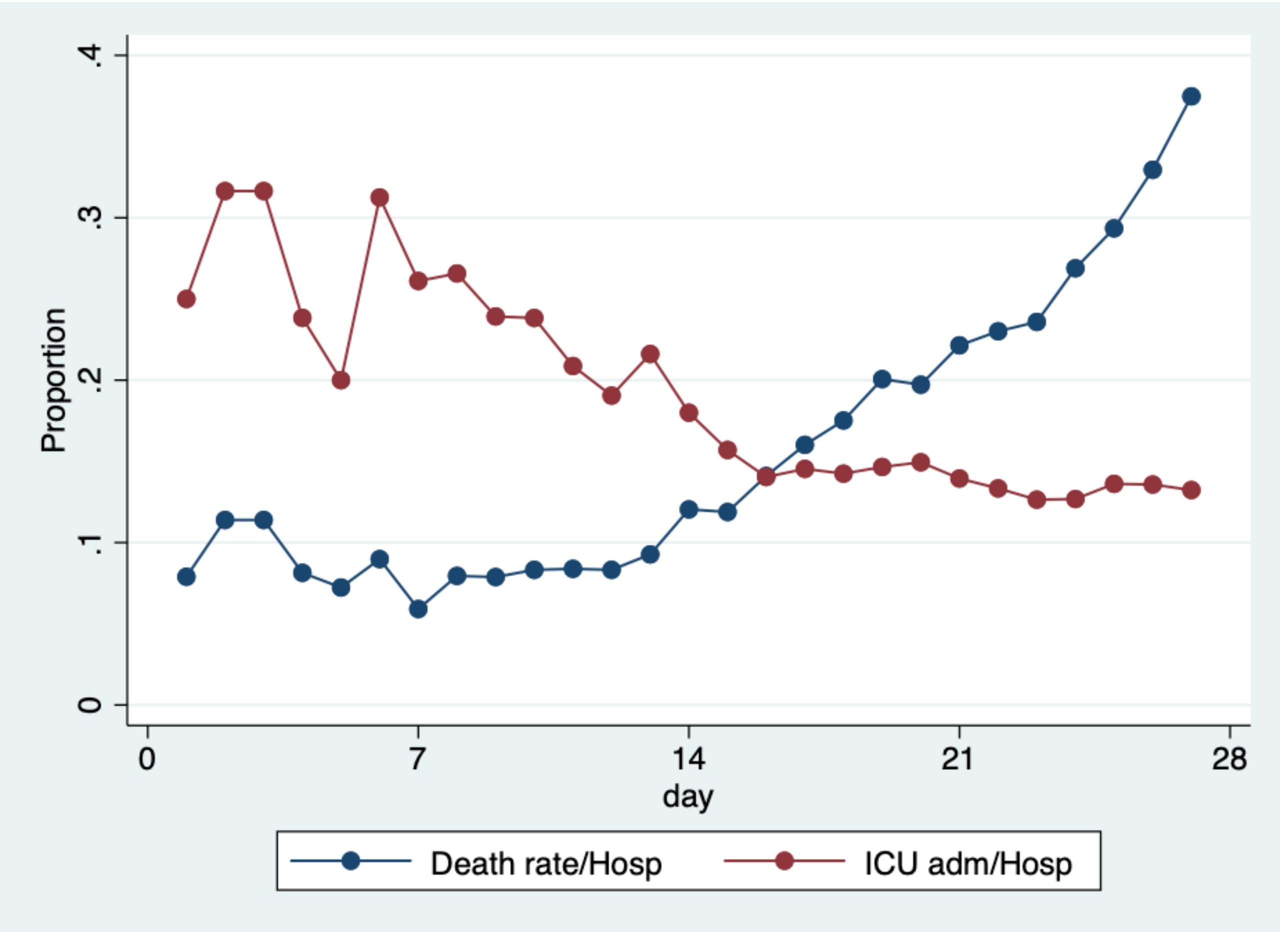
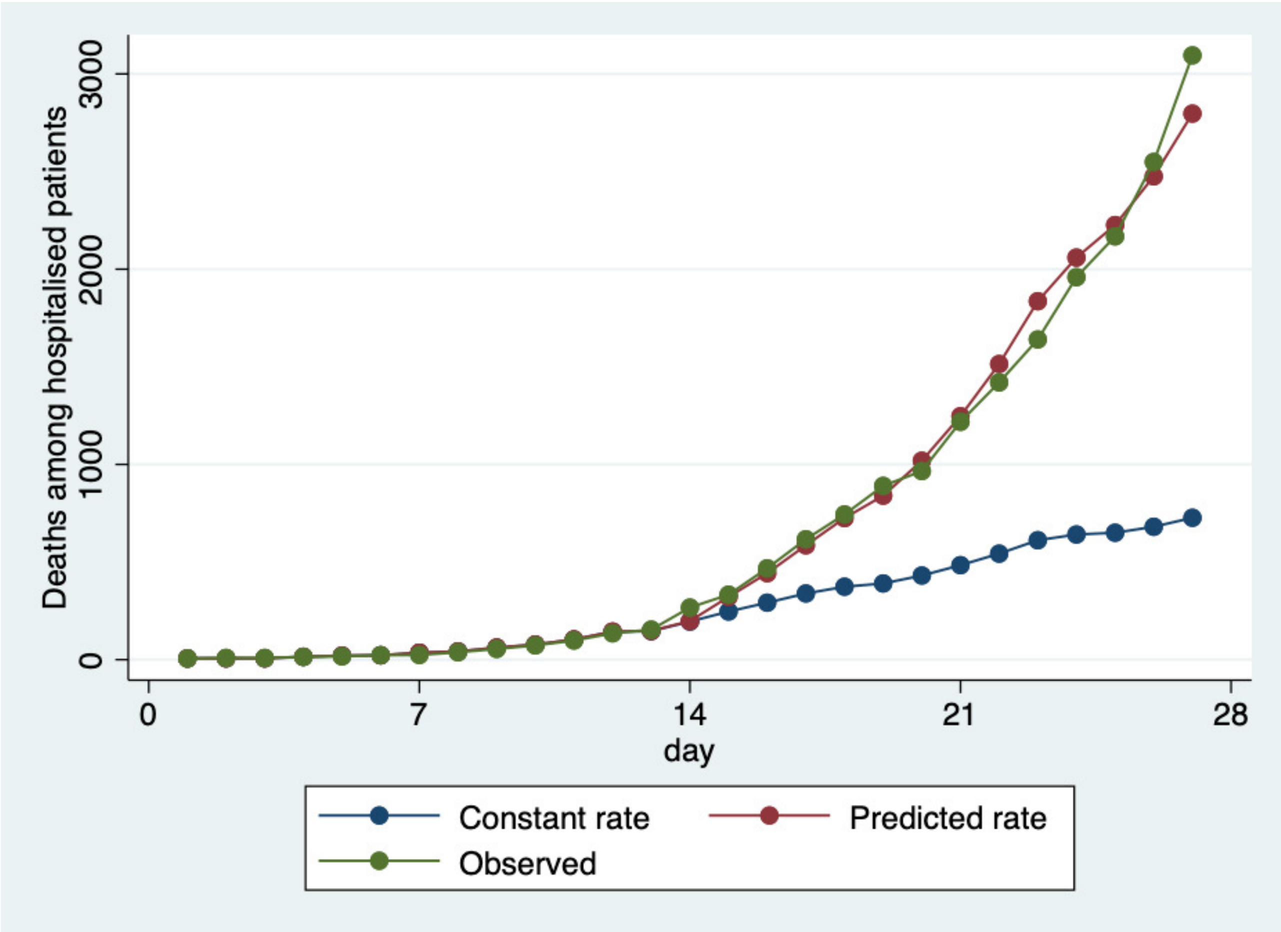


Figure 4


Deaths among hospitalised patients
0
1000
2000
3000
0
7
14
21
28
day
Constant rate
Predicted rate
Observed