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Appendix

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Appendix to “Towards a theoretically-informed policy against a rakghoul plague outbreak”

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S1 A simple population dynamics model of humans and rakghouls

The model is based on the Lotka-Volterra pair of differential equations[1]:

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \alpha \cdot H - \frac{\beta_1 \cdot R \cdot H}{1 + \beta_1 \cdot t_\mathrm{h} \cdot \zeta \cdot H + \delta \cdot \beta_2 \cdot t_\mathrm{h} \cdot R}, \tag{1}$$

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{\beta_1 \cdot R \cdot H \cdot [\epsilon_1 \cdot \zeta + (1-\zeta)] + \delta \cdot \beta_2 \cdot R^2 \cdot (\epsilon_2 - 1)}{1 + \beta_1 \cdot t_\mathrm{h} \cdot \zeta \cdot H + \delta \cdot \beta_2 \cdot t_\mathrm{h} \cdot R} - \theta \cdot R. \tag{2}$$

Here H and R represent the number of humans and rakghouls accordingly. α is the per capita growth rate of the human population (day^{-1}), and θ is the per capita death rate of the rakghoul population due to natural causes (day^{-1}). β_1 is the rate at which a single rakghoul searches for a human per day, whereas β_2 is the equivalent rate of rakghouls for members of their species. The model contains a Holling type II functional response[2], which means that rakghouls cannot search for food in a constant manner, but are limited by the time it takes them to consume a killed prey (t_h). ζ is the proportion of humans that are killed by a rakghoul after an encounter, while $1 - \zeta$ is the proportion that manages to escape with only a scratch or a bite. Because of that, the latter are infected with the rakghoul plague and will transform into rakghouls in a negligible amount of time. ϵ_1 and ϵ_2 stand for the efficiency of conversion of consumed humans and rakghouls respectively into new rakghoul individuals through reproduction. Lastly, δ is a parameter that controls whether rakghouls only target humans as prey or also commit cannibalism. According to the principle of optimal foraging[3–5], a predator will choose among available prey in a manner that maximises its energy intake rate. Hence, in this model, rakghouls will become cannibalistic only if the energy obtained by eating humans is less than or equal to the energy that would be obtained through cannibalism. Solving

$$\frac{\beta_1 \cdot R \cdot H \cdot \epsilon_1 \cdot \zeta}{1 + \beta_1 \cdot t_\mathrm{h} \cdot \zeta \cdot H} = \frac{\beta_1 \cdot R \cdot H \cdot \epsilon_1 \cdot \zeta + \beta_2 \cdot R^2 \cdot \epsilon_2}{1 + \beta_1 \cdot t_\mathrm{h} \cdot \zeta \cdot H + \beta_2 \cdot t_\mathrm{h} \cdot R} \tag{3}$$

for H gives the critical number of humans ($\widehat{H}$) at or below which rakghouls also become cannibalistic:

$$\widehat{H} = \frac{\epsilon_2}{\beta_1 \cdot t_\mathrm{h} \cdot \zeta \cdot (\epsilon_1 - \epsilon_2)}. \tag{4}$$

Therefore, if $H > \widehat{H}$, $\delta = 0$ (no cannibalism), and if $H \leq \widehat{H}$, $\delta = 1$. The implicit assumption is that $\epsilon_1 > \epsilon_2$ for humans to be preferred to rakghouls when the former are available in high enough numbers.

The model makes a number of assumptions that should be addressed in future work. First, the model is not spatially explicit and assumes that humans and rakghouls are evenly distributed in the area of study. Rakghouls feed exclusively on humans and/or other rakghoul individuals, and not on other animals or plants. Humans are unable to kill rakghouls, which should inexorably lead to the extinction of the local human population, as long as no other action is taken and the values of other parameters (e.g., the rakghoul death rate) are biologically realistic. The time it takes a rakghoul to consume prey is independent of the prey type (human or rakghoul). Finally, the transformation of a human into a rakghoul has no effect on rakghoul death rate.

S1.1 Parameterisation and numerical solutions

To constrain the parameter space of the model, some parameters were fixed to constant realistic values or value ranges based on published measurements for analogous systems, where available. In particular, the fraction of humans that are killed after a rakghoul attack, ζ, was set to a seemingly realistic value of 0.9, as very few individuals have been reported to have escaped after an encounter with a rakghoul[6,7]. The growth rate of the human population and the death rate of the rakghoul population – which is considered equal to the human death rate – were based on the 2016 official statistics for New South Wales[8,9]. More specifically, the value for the human growth rate was calculated as the difference between the birth rate and the death rate, and thus accounts for the entire population and not only for individuals that are reproductively active. β_1 and β_2 were arbitrarily assigned values between 4 and 10, whereas t_h was varied from 0.02 days ($\sim$ 29 minutes) to 0.12 days ($\sim$ 3 hours). To constrain ϵ_1 and ϵ_2 to ecologically realistic values, we used a study of population dynamics of elks and wolves[10], in which the median conversion efficiency was estimated at 0.07, with a 95% confidence interval from 0.031 to 0.124. We used a similar range of values for both parameters, i.e. from 0.02 to 0.2, with the condition that $\epsilon_1 - \epsilon_2 > 0.005$, to satisfy the optimal foraging assumption that humans are preferred to rakghouls as prey because they provide a greater amount of energy to the predator.

Numerical solutions for all possible parameter combinations were obtained using the Dormand-Prince method[11], as implemented in the SciPy library[12]. More specifically, the ordinary differential equations were integrated until both humans and rakghouls became extinct (in the latter case, due to cannibalism). The initial population size for humans was set to 7,725,900 (the estimated population size of New South Wales in 2016)[8], whereas the initial number of rakghouls was set to 2. To avoid mathematical artefacts, when the number of rakghoul or human individuals was reduced to a decimal below one, it was first set to zero before continuing the integration.

Fig. S1 provides a summary of all obtained numerical solutions, highlighting correlations between parameters of the model and human/rakghoul extinction times. The most influential parameter appears to be t_h, whereas it is noteworthy that there is a strong correlation in the time to extinction between humans and rakghouls. This suggests that the cannibalistic behaviour of rakghouls – when humans are not present – could be exploited to control and eliminate the plague.

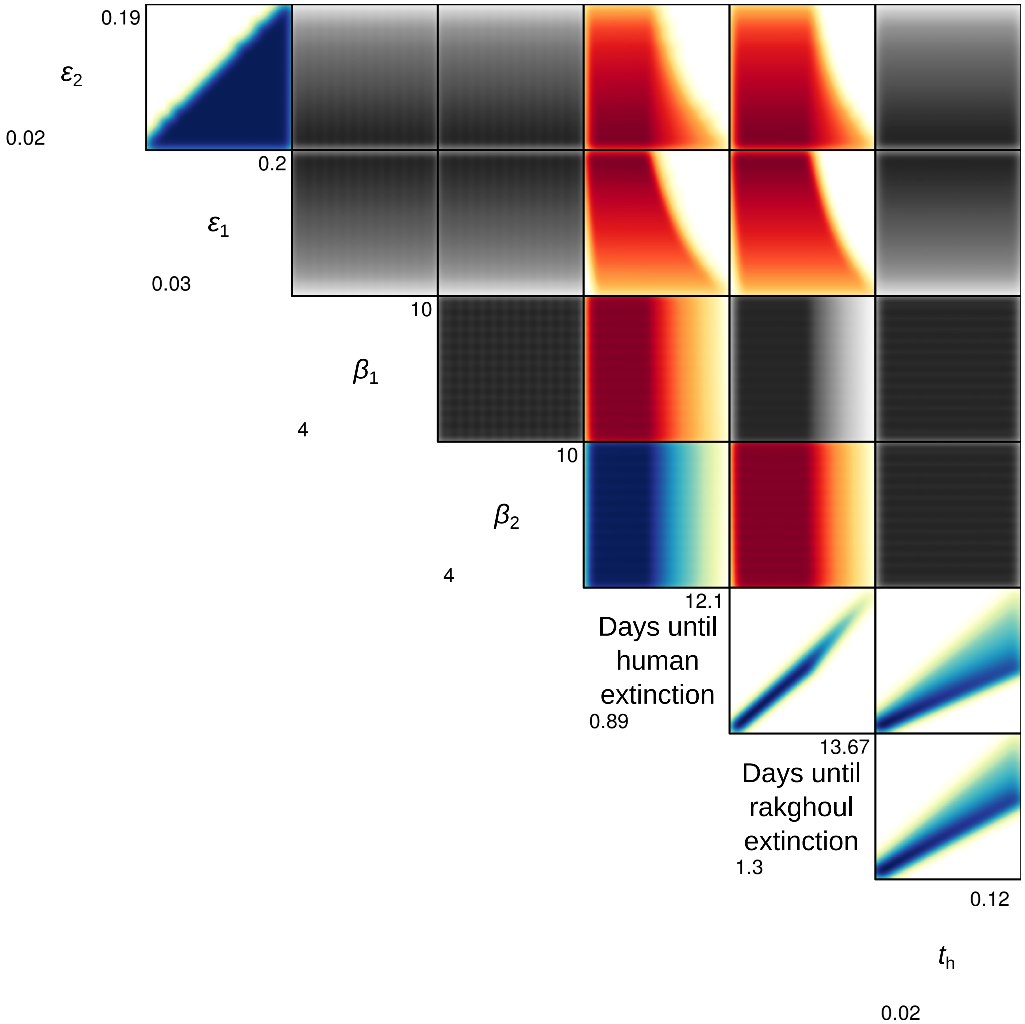

Figure S1: Correlation matrix of model parameters along with the time periods until humans/rakghouls become extinct. Diagonal panels show the variable name or symbol along with its minimum and maximum values, whereas off-diagonal panels display pairwise scatter plots of the variables. The colour of each plot reveals the statistical correlation between the two variables, with blue/red/grey data points representing positive/negative/non-statistically significant correlations respectively. To correct for multiple tests of statistical significance, p-values were adjusted using the Holm method[13]. The colour depth is proportional to the density of overlapping data points in each area of the graph. Overall, the variables that correlate most strongly with the number of days until human/rakghoul extinction are t_{h} and ϵ_1. However, the tightest correlation in this matrix is that between the time to human extinction and the time to rakghoul extinction. This leads to the conclusion that the extinction of rakghouls due to cannibalism would always take place shorty after the extinction of the local human population.

S2 A variant of the model in which humans are evacuated

To understand if an evacuation of the human population could be a practical and efficient strategy for controlling the spread of the disease, we simplified the model by removing the parameters that are relevant to the human population:

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{\beta_2 \cdot R^2 \cdot (\epsilon_2 - 1)}{1 + \beta_2 \cdot t_\mathrm{h} \cdot R} - \theta \cdot R. \tag{5}$$

Note that here $\delta = 1$, given that only rakghoul individuals are available.

Numerical solutions for this differential equation were obtained as in the previous section, with the only difference that the initial number of rakghouls ranged from 500 to 2,000,000. This variation in initial population size reflected the activation of the evacuation strategy at different time points of the outbreak (i.e., from early to very late). The results are shown in fig. S2.

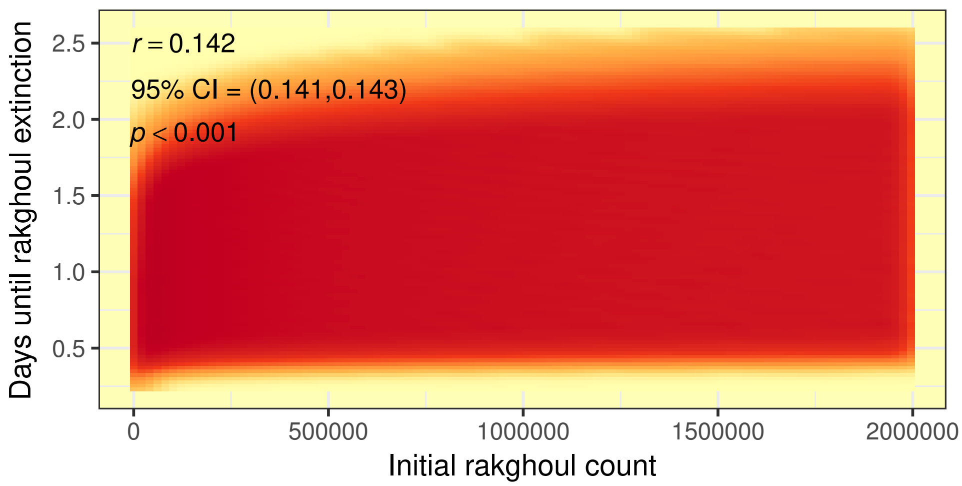

Figure S2: Estimated time for rakghoul extinction in the absence of humans, as a function of initial rakghoul count. The colour depth reflects the density of data points at each position of the graph. More precisely, many overlapping data points can be found in dark red areas, whereas areas with very few or no data points are coloured yellow. The very weak correlation indicates that the size of the initial rakghoul population has only a minor effect on their time to extinction, which is expected to occur within about two and a half days at most.

S3 Extending the original model to include military action

The spread of the disease could alternatively be prevented through extermination of rakghouls by the military via armed melee combat. To model this, equation (1) – that describes the dynamics of the human population – was paired with the following equation for the rakghoul population:

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{\beta_1 \cdot R \cdot H \cdot [\epsilon_1 \cdot \zeta + (1-\zeta)] + \delta \cdot \beta_2 \cdot R^2 \cdot (\epsilon_2 - 1)}{1 + \beta_1 \cdot t_\mathrm{h} \cdot \zeta \cdot H + \delta \cdot \beta_2 \cdot t_\mathrm{h} \cdot R} - \theta \cdot R - \beta_3 \cdot \psi \cdot H \cdot R. \tag{6}$$

The extra parameters introduced here are β_3 and ψ. β_3 is the rate at which a single soldier searches for and kills a rakghoul per day, while ψ is the proportion of soldiers in the human population. An extra assumption for this model variant is that when a soldier detects a rakghoul first, the latter will always be killed without managing to inflict any injury on the former. ψ was set to ~ 0.0029, based on the number of Australian Defence Force employees in the state of New South Wales[14], and was held constant. To account for a delay between the time of rakghoul invasion and the time of military involvement (t_resp), β_3 was parameterised as follows:

$$\begin{cases} \beta_3 = 0, & \text{if } t < t_\mathrm{resp} \\ \beta_3 \in [0.005, 0.095], & \text{otherwise.} \end{cases} \tag{7}$$

We deliberately set very low values to β_3 – in comparison to β_1 and β_2 – to account for a worst-case scenario of low military effectiveness against rakghouls. t_resp was allowed to vary between 2 and 24 hours.

The two equations were numerically integrated until the extinction of either rakghouls or humans. To analyse the dataset of numerical solutions, we constructed a decision tree using the rpart R package[15]. This machine learning algorithm tries to predict an outcome (human extinction versus rakghoul eradication) through combinations of other predictor variables (the varying parameters of equations (1) and (6)), in a tree-like manner. To this end, the numerical solutions were split between a training dataset (75% of the original dataset; used for training the decision tree) and a testing dataset (25% of the original dataset; used for evaluating the tree quality). The frequencies of human extinction and rakghoul elimination were kept constant across the two datasets.

The resulting tree is shown in Box 3, in the main article.

References

[1] Otto SP, Day T. A biologist's guide to mathematical modeling in ecology and evolution. Princeton, New Jersey: Princeton University Press; 2007.

[2] Holling CS. Some characteristics of simple types of predation and parasitism. *Can Entomol.* 1959;91(7):385–398.

[3] Charnov EL, Orians GH. Optimal foraging: some theoretical explorations; 1973. http://hdl.handle.net/1928/1649 (accessed July 2017).

[4] Charnov EL. Optimal foraging, the marginal value theorem. *Theor Popul Biol.* 1976;9(2):129–136.

[5] Stephens DW, Krebs JR. Foraging theory. Princeton, New Jersey: Princeton University Press; 1986.

[6] Miller JJ, Harrison M, Hepburn S, et al. Star Wars: Vector Volume 1. Milwaukie, Oregon: Dark Horse Comics; 2009.

[7] Williams R, Ostrander J, Weaver D, et al. Star Wars: Vector Volume 2. Milwaukie, Oregon: Dark Horse Comics; 2009.

[8] Australian Bureau of Statistics. 3101.0 – Australian Demographic Statistics; June 2016. http://www.abs.gov.au/ausstats/abs@.nsf/mf/3101.0 (accessed Feb. 2017).

[9] Australian Bureau of Statistics. Deaths, Year of registration, Summary data, Sex, States, Territories and Australia; 2016. http://stat.data.abs.gov.au/Index.aspx?Queryid=456 (accessed Feb. 2017).

[10] Goldberg JF, Hebblewhite M, Bardsley J. Consequences of a refuge for the predator-prey dynamics of a wolf-elk system in Banff National Park, Alberta, Canada. *PLoS One.* 2014;9(3):e91417.

[11] Dormand JR, Prince PJ. A family of embedded Runge-Kutta formulae. *J Comput Appl Math.* 1980;6(1):19–26.

[12] Jones E, Oliphant E, Peterson P, et al. SciPy: Open Source Scientific Tools for Python; 2001-. http://www.scipy.org (accessed June 2017).

[13] Holm S. A simple sequentially rejective multiple test procedure. *Scand J Stat.* 1979;6(2):65–70.

[14] Australian Government, Department of Defence. Defence Annual Report 2015-16. Volume One: Performance, governance and accountability; 2016. http://www.defence.gov.au/annualreports/15-16/Downloads/DAR_2015-16_Vol1.pdf (accessed Feb. 2017).

[15] Therneau T, Atkinson B, Ripley B. rpart: Recursive Partitioning and Regression Trees; 2014. R package version 4.1-8. http://cran.r-project.org/package=rpart (accessed June 2017).